Student Number: ____________

Teacher: ____________

St Catherine's School

Waverley, Sydney

Year 12 Extension 1 Mathematics

Trial Examination

Task #4

August 2009

Time allowed: *2 hours*

Reading time: *5 minutes*

General Instructions

- Working time – 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used.
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value
- Start each question on a new page
- Start Question 5 in a new book

The following list of standard integrals may be used:

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, n \neq -1; x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\ dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\ dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\ dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\ dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\ dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\ dx = \sin^{-1}\frac{x}{a},\ a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\ dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\ dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Question 1 – 12 Marks **Marks**

a) Solve for x: $\frac{5}{x-1} \leq 3$ **3**

b) The point $\boldsymbol{P}(1,3)$ divides the join of the points $A(a,b)$ and $\boldsymbol{B}(0,5)$ externally in the ratio of 2 : 1. Find the values of a and b. **2**

c) Find $\lim_{x \to 0} \frac{\sin 5x}{2x}$ **2**

d) Find the acute angle, to the nearest degree, between the curves $y = \sin x$ and $y = \cos x$ at the point of intersection $(\frac{\pi}{4}, \frac{1}{\sqrt{2}})$ **3**

e) Find $\frac{d}{dx}(x \ \sin^2 x)$ **2**

Question 2 – 12 Marks *START A NEW PAGE* **Marks**

a) If $x = 1$ and $x = 2$ are the roots of the polynomial:

$$ax^3 + bx^2 + 11x - 6 = 0$$

(i) Find a and b **2**

(ii) Hence find the third root. **1**

b) Given the function $f(x) = \sqrt{x+6}$

(i) Write the inverse function and state its domain **2**

(ii) Find the x coordinates of the point(s) of intersection of the given function and its inverse. **2**

c) There are 4 girls and 4 boys. Find the number of ways:

(i) They can line up if the girls want to stay together. **1.5**

(ii) A committee of 5 people can be formed if there are at least 3 girls in the committee **2**

(iii) They can sit in a circle if the boys and girls alternate. **1.5**

Question 3 – 12 Marks *START A NEW PAGE* **Marks**

a) Consider the function $f(x)=\dfrac{x-1}{4-x^2}$

(i) State the domain **1**

(ii) Find $\lim\limits_{x\to\infty} f(x)$ **1**

(iii) Sketch $y=f(x)$, clearly showing the x and y intercepts. **3**

b) Given that $y=xe^x$, show using mathematical induction that:

$$\frac{d^n y}{dx^n}=ne^x+xe^x, \text{ for all } n\geq 1.$$ **4**

c) If $\tan^{-1}3+\tan^{-1}x=\dfrac{\pi}{4}$, find the value of x. **3**

Question 4 – 12 Marks *START A NEW PAGE* **Marks**

a) (i) Differentiate: $x\tan^{-1}x$ **1**

(ii) Hence evaluate: $\int_0^1 \tan^{-1}x\ dx$ **2**

b) Consider the parabola: $x^2=4y$. $P(2p,p^2)$ and $Q(2q,q^2)$ are two points on this parabola.

(i) Assume that the equation of the tangent at P is given by: $y=px-p^2$. Hence or otherwise show that the coordinates of T, the point of intersection of the tangents at P and Q is: $(p+q,pq)$ **2**

(ii) Show that $SP=p^2+1$, where S is the focus of the parabola **2**

(iii) Find the locus of T if P and Q move such that $SP+SQ=4$. **2**

c) In the expansion of $(1+2x)^n$, the ratio of the coefficients of x^5 and x^4 is 2:1.

Find the value of n. **3**

Question 5 – 12 Marks *START A NEW BOOKLET* **Marks**

a) Consider the expansion of $(2+3x)^{11}$ in the ascending powers of x:

(i) Write down T_{r+1} and T_r, where T_r is the r^{th} term — 1

(ii) Show that the $\dfrac{coefficient\ \ of\ \ T_{r+1}}{coefficient\ \ of\ \ T_r}=\dfrac{36-3r}{2r}$ — 2

(iii) Hence find the greatest coefficient of $(2+3x)^{11}$ — 2

b) Find the term independent of x in the expansion of $(\dfrac{2}{x}+3x)^{12}$ — 3

c) Evaluate $\int_0^2 \sqrt{4-x^2}\,dx$ using the substitution $x=2\ \cos\theta$ — 4

Question 6 – 12 Marks *START A NEW PAGE* **Marks**

a) Find $\int \dfrac{1}{1+4x^2}dx$ — 2

b) The acceleration of a particle moving in SHM is given by: $\dfrac{d^2x}{dt^2}=-4(x-3)$. If initially the particle starts at $x=4$ with a velocity of 2 cm/sec, write the equation of motion in the form $x=\underset{\sim}{a}+\underset{\sim}{b}\cos(\underset{\sim}{n}t+\underset{\sim}{\varepsilon})$. — 5

c) (i) Solve for x: $0\le\sqrt{x-1}\le1$ — 2

(ii) State the Domain and Range of the function $y=\sin^{-1}\sqrt{x-1}$ — 2

(iii) Sketch the function $y=\sin^{-1}\sqrt{x-1}$ — 1

Question 7 – 12 Marks *START A NEW PAGE* **Marks**

a) (i) Show that $\cos x - \sin x = \sqrt{2}\cos(x + \frac{\pi}{4})$ **2**

(ii) Find the general solution of the equation: $\cos x - \sin x = 1$ **2**

(iii) Sketch the function $y = \cos x - \sin x$ ~~in the domain $0 \le x \le 2\pi$~~.
Label clearly all the main features: the x intercepts, the turning points, the end points

3

b) A cat sits on top of a wall which is 5 metres above the ground. It sees a mouse on the ground, which is exactly 4 metres from the wall.

The cat jumps horizontally from the top of the wall with an initial velocity of 6 metres per second.

Assume acceleration due to gravity is 10 metres per sec^2. Neglect air resistance and assume that the motion of the cat is in projectile motion.

Placing the axes at the bottom of the wall, the equations of motion are given by

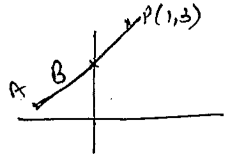
$$\frac{dx}{dt} = 6,\ x = 6t,\ \frac{dy}{dt} = -10t,\ y = -5t^2 + 5$$

(i) Find the time taken for the cat to reach the ground **1**

(ii) Just as the cat starts to jump, the mouse runs away from the wall at the rate of 4 metres per second. Find the distance between the cat and the mouse when the cat lands on the ground. **2**

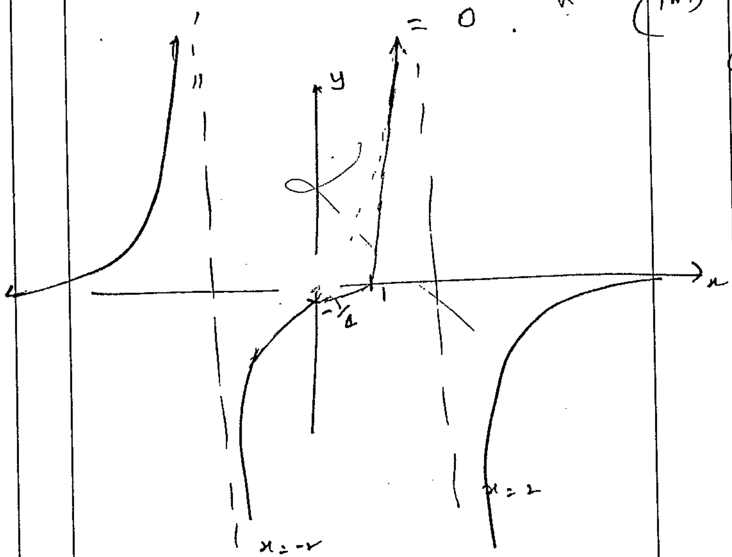
(iii) Find the magnitude and the direction of the velocity with which the cat lands on the ground. **2**

END OF PAPER

Qn	Solutions	Marks	Comments: Criteria
1.	$\frac{5}{x-1} \le 3$; $x \neq 1$		
	$5(x-1)^2 \cdot \frac{1}{x-1} \le 3(x-1)^2$	1	
	$5(x-1) \le 3(x-1)^2$		
	$(x-1)(5-3(x-1)) \le 0$		
	$(x-1)(8-3x) \le 0$ 	1	
	$x < 1$ or $x \ge \frac{8}{3}$	1	
b)	 A : (a, 3) +2 : -1 B : (0,1) P : (1,3)		
	$1 = -\cancel{a}$; $3 = 10 - b$. $b = 7$		1 for a 1 for b
	∴ A : (−1, 7)		
c)	$\lim_{x \to 0} \frac{\sin 5x}{2x}$		
	$= \lim_{5x \to 0} \frac{\sin 5x}{5x} \cdot \frac{5}{2}$		
	$= \frac{5}{2} \times \lim_{5x \to 0} \frac{\sin 5x}{5x}$	1m	
	$= \frac{5}{2} \times 1$	1m.	
	$= \frac{5}{2}$		

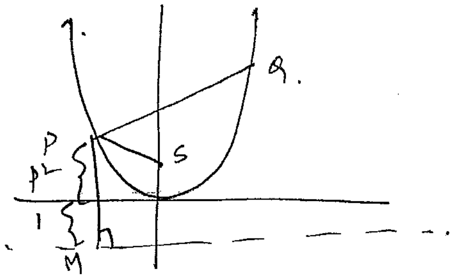
Qn	Solutions	Marks	Comments: Criteria
d)	$y = \sin x$ $\quad$ $y = \cos x$		
	$\frac{dy}{dx} = \cos x$ $\quad$ $\frac{dy}{dx} = -\sin x$		
	$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \cos\frac{\pi}{4}$ $\quad$ $\left(\frac{dy}{dx}\right)_{x=\pi/4} = -\sin\frac{\pi}{4}$		
	$= \frac{1}{\sqrt{2}}$ $\quad$ $= -\frac{1}{\sqrt{2}}$	$(1\frac{1}{2})$	
	If θ is the acute angle between the curve, it is the angle between the tangents at the point of intersection	$(\frac{1}{2}m)$	for formula.
	$\tan\theta = \left\| \frac{\frac{1}{\sqrt{2}} - (-\frac{1}{\sqrt{2}})}{1 - \frac{1}{2}} \right\|$		
	$\theta = \tan^{-1} 2\sqrt{2}$ $= 71^\circ$ to nearest degree.	(1m)	$(-\frac{1}{2})$ not nearest degree
e)	$\frac{d}{dx}(x \sin^2 x)$		
	$= x.(2\sin x \cos x) + \sin^2 x$	2m.	
	$= \sin x(2x\cos x + \sin x)$		
Q. 2 a)	$ax^3 + bx^2 + 11x - b = 0$		
	$x=1$ is a root ∴ $a + b + 5 = 0$	— ①	✗
	$x = 2$ is a root $\quad$ $8a + 4b + 16 = 0$ $2a + b + \cancel{4} = 0$	— ②	✗
	$a + b = -5$ $2a + b = -\cancel{4}$ $a = +\cancel{1}$ (1m) $b = -\cancel{6}$ (1m)		
	if α is ~~lastroot~~ third root; $1 + 2 + \alpha = -\cancel{6}$ $\alpha = -\frac{\cancel{8}\,6}{3} - 3 = \frac{\cancel{8}}{3}$	(-1m)	

Qn	Solutions	Marks	Comments: Criteria
Q.2 b)	$y = \sqrt{x+6}$ $x = \sqrt{y+6}$ $x^2 = y+6$ $\quad \underline{y = x^2 - 6}$; Domain of $y = x^2-6$ is ~~all~~ x ; but as an inv. fn. of $y=\sqrt{x+6}$; which is the range of $y=\sqrt{x+6}$; which is $y \geq 0$ ∴ Domain of $y = x^2-6$; is restricted to $x \geq 0$.		$-\frac{1}{2}$ m if not restricted
	(ii) $y = \sqrt{x+6}$ and its inverse function $y = x^2-6$ $(x \geq 0)$ meet on $y = x$ $x^2 - 6 = x$ $x^2 - x - 6 = 0$ $(x-3)(x+2) = 0$ $x = 3;\ x = -2$ $(x \neq -2)$ ∴ The point of intersection is $(3, 3)$		$-\frac{1}{2}$ m if $(-2,-2)$ is not eliminated.
c)	$\boxed{GGGG}$ ~~A~~ B. (i) $5! \times 4!$ $\quad (4! \times 5 \times 4!) = 2880$ (iii) $3! \times 4!$ ways.	$(-\frac{1}{2}$ m) $-(\frac{1}{2}$ m)	if $4! \times 4!$ if doubled
	(ii). 3 girls or 4 girls ${}^4C_3 \times {}^4C_2 + {}^4C_4 \times {}^4C_1$ $= 24 + 4$ $= 28$.		

Qn	Solutions	Marks	Comments: Criteria
Q.3 a)	$f(x) = \dfrac{x-1}{4-x^2}$ Domain: $x \neq 2,\ x \neq -2$	(1m)	
	$\lim\limits_{x\to\infty} f(x) = \dfrac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{4}{x^2} - \frac{x^2}{x^2}}$ $= 0$.	(1m)	
			(A) The 3 asymptotes $1\frac{1}{2}$ m (B). Correct signs $1\frac{1}{2}$ m. -0.5 for wrong y-intercept.
	Sign + $\quad -2 \quad$ 1+ $\quad 2 \quad$ − $\frac{-3-1}{4-9}$ $\quad \frac{0-1}{4-0}$ $\quad \frac{3-1}{4-9}$		

Qn	Solutions	Marks	Comments: Criteria
b)	$y = xe^x$ —— ⊗		
	Let P_n: $\dfrac{d^n y}{dx^n} = ne^x + xe^x$		
	P_1: $\dfrac{dy}{dx} = xe^x + e^x$ (differentiating ⊗)		
	$\therefore P_1$ is true	(1m)	
	Let P_k be true $\dfrac{d^k y}{dx^k} = ke^x + xe^x$ —— (A)	($\frac{1}{2}$m)	
	To show $\dfrac{d^{k+1}y}{dx^{k+1}} = (k+1)e^x + xe^x$.		
	$\dfrac{d^{k+1}y}{dx^{k+1}} = \dfrac{d}{dx}\left(ke^x + xe^x\right)$	($\frac{1}{2}$m)	
	$= ke^x + xe^x + e^x$ using (A)	($\frac{1}{2}$m)	
	$= (k+1)e^x + xe^x$	($\frac{1}{2}$m)	
	$\therefore P(k+1)$ is true if $P(k)$ is true		
	$P(1)$ is true; $P(k+1)$ is true if $P(k)$ is true $\therefore$ By the principle of mathematical induction $P(n)$ is true for all $n \geq 1$.	($\frac{1}{2}$m)	
c)	Let $\tan^{-1}3 = y$; $\tan^{-1}x = k$		
	$\therefore \tan y = 3$; $\tan k = x$		
	Consider $\tan(\tan^{-1}3 + \tan^{-1}x) = \tan \pi/4$		
	$\Rightarrow \tan(y+k) = 1$		

Qn	Solutions	Marks	Comments: Criteria
	$\Rightarrow \dfrac{\tan y + \tan k}{1 - \tan y \tan k} = 1$	✓	
	$\Rightarrow \dfrac{3+x}{1-3x} = 1$	✓	
	$3 + x = 1 - 3x$		
	$4x = -2$		
	$x = -\frac{1}{2}$.	✓	
Q.4	$y = x\tan^{-1}x$		
	$\dfrac{dy}{dx} = x \cdot \dfrac{1}{1+x^2} + \tan^{-1}x$	(1m)	
	$\therefore \int\left(\dfrac{x}{1+x^2} + \tan^{-1}x\right)dx = x\tan^{-1}x$		($\frac{1}{2}$m)
	$\therefore \int_0^1 \tan^{-1}x\,dx = x\tan^{-1}x - \int \dfrac{x}{1+x^2}dx$		($\frac{1}{2}$m)
	$\left(x\tan^{-1}x\right)_0^1 - \frac{1}{2}\left(\ln(1+x^2)\right)_0^1$		($\frac{1}{2}$m)
	$= \frac{\pi}{4} - \frac{1}{2}\left[\ln 2 - \ln 1\right]$	($\frac{1}{2}$m)	
	$= \frac{\pi}{4} - \frac{1}{2}\ln 2$.		

Qn	Solutions	Marks	Comments: Criteria
b)			
	$y = px - p^2$ tgt at P — ①		
	$\therefore\ y = qx - q^2$ tgt at Q — ②		
	① and ② meet at	$\checkmark$	
	$px - p^2 = qx - q^2$		
	$(p-q)x = p^2 - q^2$		
	$x = \dfrac{(p-q)(p+q)}{p-q}$		
	$x = p+q \quad (p \neq q)$		
	Sub in ①		
	$y = p(p+q) - p^2$		
	$= pq$		
	$\therefore\ T: (p+q, pq)$		
(ii)	$S: (0, 1)$; directrix is $y = -1$	($\frac{1}{2}$m)	
	by def. $SP = PM$, where $PM \perp$ directrix. (1m)		or $\sqrt{(0-2p)^2+(1-p^2)^2}$
	$PM = p^2+1$.	($\frac{1}{2}$m)	$= \sqrt{4p^2+1-2p^2+p^4}$ $= \sqrt{p^4+2p^2+1}$ $= p^2+1$

Qn	Solutions	Marks	Comments: Criteria
	$SP + SQ = 4$		
	$p^2+1 + q^2+1 = 4$		
	$p^2+q^2 = 2$	(1m)	
	$T: \ x = p+q$		
	$y = pq$		
	also $p^2+q^2 = 2$		
	$(p+q)^2 = p^2+q^2+2pq$	(1m)	
	$x^2 = 2 + 2y$		
c)	$(1+2x)^n = {}^nC_0 + {}^nC_1(2x) + {}^nC_2(2x)^2 + \dots$		
	Coeff of $x^5 = {}^nC_5 (2)^5$	($\frac{1}{2}$m)	
	Coeff of $x^4 = {}^nC_4\, 2^4$	($\frac{1}{2}$m)	
	$\dfrac{{}^nC_5 \times 2^5}{{}^nC_4 \times 2^4} = \dfrac{2}{1}$	($\frac{1}{2}$m)	
	${}^nC_5 \times 2^5 = {}^nC_4 \times 2^5$		
	${}^nC_5 = {}^nC_4$	* ($1\frac{1}{2}$m)	
	$\therefore\ n = 9$.		

Qn	Solutions	Marks	Comments: Criteria

Q.5

$(2+3x)^{11} = {}^{11}C_0\, 2^{11} + {}^{11}C_1\, 2^{10}(3x) + {}^{11}C_2\, 2^{9}(3x)^2 + \dots$

$T_{r+1} = {}^{11}C_r\, 2^{11-r}(3x)^r$

$T_r = {}^{11}C_{r-1}\, 2^{11-(r-1)}(3x)^{r-1}$

$\dfrac{\text{Coefficient of } T_{r+1}}{\text{Coeff. of } T_r} = \dfrac{{}^{11}C_r\, 2^{11-r} \cdot 3^r}{{}^{11}C_{r-1}\, 2^{12-r} \cdot 3^{r-1}}$

$\dfrac{{}^{11}C_r}{{}^{11}C_{r-1}} = \dfrac{11!}{r!(11-r)!} \times \dfrac{(r-1)!(12-r)!}{11!}$

$= \dfrac{12-r}{r}$

∴ The ratio is

$\dfrac{12-r}{r} \times \dfrac{3}{2}$

Coeff of $T_{r+1} \geq$ Coeff of T_r

$\dfrac{36-3r}{\cancel{2r}} \geq 2r$

$36 \geq 5r$

$r \leq \dfrac{36}{5}$ (1m)

$r = 1, 2, \dots, 7$; Coeff of $T_{r+1} >$ Coeff of T_r

$r = 8, 9, \dots$; Coeff of $T_{r+1} <$ Coeff of T_r

(ii) Hence find the greatest coeff — must be found using $\frac{36-3r}{2r}$ if expanded; $\frac{1\frac{1}{2}}{2}$.

CT_1, CT_2, CT_8, CT_9

Qn	Solutions	Marks	Comments: Criteria

∴ Coeff of T_8 is the greatest coeff. ($\frac{1}{2}$m)

$= {}^{11}C_7 \times 2^4 \times 3^7$ ($\frac{1}{2}$m)

(11547360)

(b) $T_{r+1} = {}^{12}C_r \left(\frac{2}{x}\right)^{12-r}(3x)^r$

$= {}^{12}C_r\, 2^{12-r}\, 3^r\, x^{r-12+r}$ $\frac{1}{2}$

const. term; $2r-12=0$, $r=6$ 1m

($\frac{1}{2}$m) $T_7 = {}^{12}C_6\, 2^6\, 3^6$ $\frac{1}{2}$

c) $x = 2\cos\theta$

$dx = -2\sin\theta\, d\theta$

$x = 0$; $\theta = \frac{\pi}{2}$

$x = 2$; $\cos\theta = 1$, $\theta = 0$ (1m)

$\therefore \int_0^2 \sqrt{4-x^2}\,dx = \int_{\frac{\pi}{2}}^{0} \sqrt{4-4\cos^2\theta}\,(-2\sin\theta)\,d\theta$

$= \int_0^{\frac{\pi}{2}} (2\sin\theta)(2\sin\theta)\,d\theta$

note $\sqrt{4-4\cos^2\theta} = \sqrt{4\sin^2\theta} = |2\sin\theta| = 2\sin\theta \ (0<\theta<\frac{\pi}{2})$

$= 4\int_0^{\pi/2} \sin^2\theta\, d\theta$ (1m)

$\cos 2\theta = 1 - 2\sin^2\theta$

$= \frac{4}{2}\int_0^{\pi/2} (1-\cos 2\theta)\,d\theta$ ($\frac{1}{2}$m)

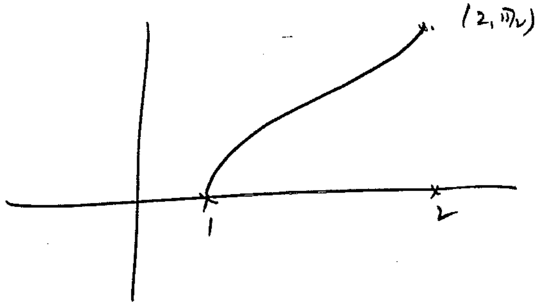
$= 2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$ ($\frac{1}{2}$m) $= 2\left[\left(\frac{\pi}{2}-0\right)-(0)\right]$

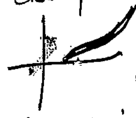
$= \pi$ ($\frac{1}{2}$m)

Qn	Solutions	Marks	Comments: Criteria
6a)	$\int \frac{1}{1+4x^2}\,dx$		$\tan^{-1}2x + c$ is $\frac{1}{2}$ out of 2 if typographical error.
	$= \int \frac{1}{4\left(\frac{1}{4}+x^2\right)}\,dx$		$-\frac{1}{2}$ for mechanical error
	$= \frac{1}{4}\cdot\frac{1}{1/2}\cdot\tan^{-1}\frac{x}{1/2} + c$		-1 for not considering $\frac{1}{a}$
	$= \frac{1}{2}\tan^{-1}2x + c$	2	
6b)	$x = a + b\cos(nt+\epsilon)$		
	$\ddot{x} = -4(x-3)$ given		
	$\therefore\ n=2;\ a=3$		
	$x = 3 + b\cos(2t+\epsilon)$	(1m)	
	$t=0;\ x=4$		
	$4 = 3 + b\cos\epsilon$		
	$b\cos\epsilon = 1.$ —— (1)	(1m)	
	$\frac{dx}{dt} = -2b\sin(2t+\epsilon)$		
	$t=0;\ v=2$ $\quad 2 = -2b\sin\epsilon$		
	$b\sin\epsilon = -1$ —— (2)	(1m)	
	from (1) & (2)		
	$\tan\epsilon = -1$		
	$\epsilon = -\pi/4$	(1m)	

Qn	Solutions	Marks	Comments: Criteria
	Sub in (1)		
	$b\cos(-\pi/4) = 1$		
	$b \times \frac{1}{\sqrt{2}} = 1$		
	$\therefore b = \sqrt{2}$	(1m)	
	Thus $x = 3 + \sqrt{2}\cos(2t-\pi/4)$		
c)	$0 \le \sqrt{x-1} \le 1$		
	$x-1 \ge 0$ also $x-1 \le 1$		
	$\therefore\ x \ge 1$ $\quad x \le 2$		
	$\therefore\ 1 \le x \le 2.$		
	$y = \sin^{-1}\sqrt{x-1}.$		
	We need domain of a $\sin^{-1}$ function to be between -1 to 1; but $\sqrt{x-1} \ge 0$; we have combining them;		
	$0 \le \sqrt{x-1} \le 1$		
	$\therefore\ 1 \le x \le 2$ from (1) —(1m)		
	Range: $\sin^{-1}0 = 0$		
	$\sin^{-1}1 = \frac{\pi}{2}.$		
	$0 \le y \le \frac{\pi}{2}$ —— (1m)		

Qn	Solutions	Marks	Comments: Criteria
(iii)	Sketch $y = \sin^{-1}\sqrt{x-1}$	(1m)	Ignore concavity change.





marks for $(1,0)$; $(2, \pi/2)$ not honoured for wrong domain / range.

Q.7

$\cos x - \sin x = R\cos(x+\alpha)$

$= R(\cos x \cos\alpha - \sin x \sin\alpha)$

$R = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$R\cos\alpha = 1$

$R\sin\alpha = 1$

$\tan\alpha = 1$

$\therefore \alpha = \pi/4$

Thus

$\cos x - \sin x = \sqrt{2}\cos(x + \pi/4)$

$\cos x - \sin x = 1$

$\sqrt{2}\cos(x+\pi/4) = 1$

$\cos(x+\pi/4) = \frac{1}{\sqrt{2}}$ — ($\frac{1}{2}$ m)

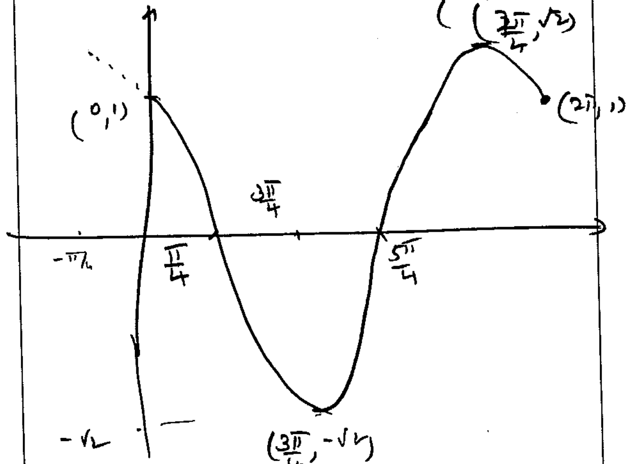
$x + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$ — (1m)

$x = 2n\pi \; ; \; 2n\pi - \frac{\pi}{2}$ — ($\frac{1}{2}$ m)

if general soln is done after $x + \frac{\pi}{4} = \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$; $x =$ only 1 out of 2

$y = \cos x - \sin x$

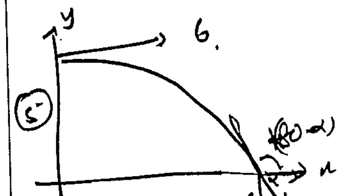
$y = \sqrt{2}\cos(x + \pi/4)$



$(\frac{\pi}{4}, 0)$	$\frac{1}{2}$ m
$(\frac{3\pi}{4}, 0)$	"
$(\frac{5\pi}{4}, -\sqrt{2})$	$\frac{1}{2}$
$(\frac{7\pi}{4}, \sqrt{2})$	$\frac{1}{2}$
$(0, 1)$	$\frac{1}{2}$
$(2\pi, 1)$	$\frac{1}{2}$

b) $\frac{dx}{dt} = 6$ $\qquad \frac{dy}{dt} = -10t$

$x = 6t$ $\qquad y = -5t^2 + 5$



when the car reaches the ground, $y = 0$

$-5t^2 + 5 = 0$

$t = 1$

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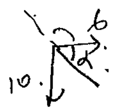
Qn	Solutions	Marks	Comments: Criteria
Q.7 (ii).	In 1 sec; the mouse travels 4 m/s. $\therefore$ it is 8 m from the wall. $x = 6 \times 1 = 6$m ; dist travelled by the car. Hence the dist. between them is 2m. $v^2 = \dot{x}^2 + \dot{y}^2$; $\dot{x} = 6$, $\dot{y} = -10$. $= 36 + 100$ $v = \sqrt{136}$ if α is as shown  $\tan\alpha = \frac{10}{6}$ $\alpha = 59^\circ 2'$ $\therefore$ The direction of the angle motion is $180 - 59^\circ 2'$ $= 120^\circ 58'$ or $-59^\circ 2'$.	(1m) (1m) (1m)	$1\frac{1}{2}/2$ if mouse is not $x = 6$